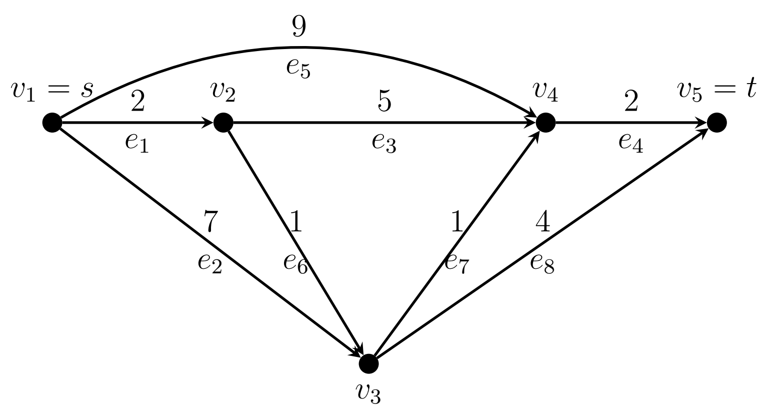
Math 482 **HW6** Section: __________ Name: ______________________________

Due Friday, March 15, 2015

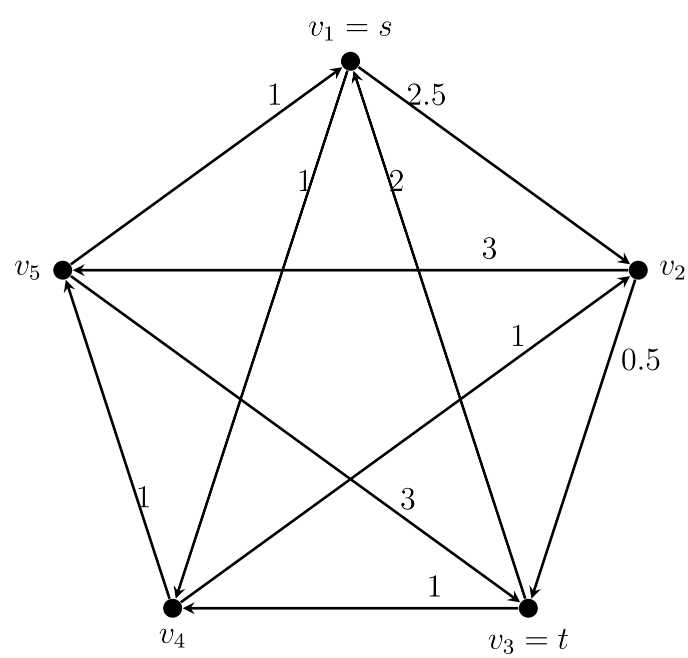
Students in section D13 (three credit hours) need to solve any four of the following five problems. Students in section D14 (four credit hours) must solve all five problems.

1. Interpreting the numbers on edges as edge lengths, solve the shortest (s, t)-path problem for the graph drawn below using the simplex method with Bland's pivot rules with the initial basis $\{e_1, e_2, e_3, e_4\}$. (See example 3.7 in the book).



2. State the dual to the shortest path problem above and use your solution to problem 1 and complementary slackness to give its solution.

3. Let G be the network with the flow drawn below with $s = v_1$ and $t = v_3$. Write the flow as a sum combination of positive flows along cycles, (s, t)-paths and (t, s)-paths.



4. Use the revised simplex method to find an optimal solution to the problem

$$\text{Minimize } z = x_1 + x_2 + x_3$$

subject to

$$\begin{cases} x_1 + x_4 - 2x_6 &= 5, \\ x_2 + 2x_4 - 3x_5 + x_6 &= 3, \\ x_3 + 2x_4 - 3x_5 + 6x_6 &= 5, \\ x_1, \ldots, x_6 &\geq 0. \end{cases}$$

5. Let A be an $m \times n$ matrix of rank m and let P be the following LP in standard form.

$$\text{Minimize } z = c^T x$$

$$\text{subject to } \begin{cases} Ax = b \\ x \geq 0 \end{cases}$$

Prove that if the P has an optimal solution and P has no degenerate optimal solutions, then there is a unique optimal solution to the dual of P. (Hint: Use the complementary slackness condition and the fact that if an LP in standard form has an optimal solution, then it has an optimal basic feasible solution)