Math 482 **HW5** Section: ____________ Name: ______________________________________

Due Friday, October 9, 2015

Students in section D13 (three credit hours) need to solve any four of the following five problems. Students in section D14 (four credit hours) must solve all five problems.

1. Use the complementary slackness condition to check whether the vector $[3,\,-1,\,0,\,2]^T$ is an optimal solution to the problem

$$\text{Maximize } z = 6x_1 + x_2 - x_3 - x_4$$

subject to

$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 & \leq & 5, \\ 3x_1 + x_2 - x_3 & \leq & 8, \\ x_2 + x_3 + x_4 & = & 1, \\ x_3, x_4 & \geq & 0. \end{cases}$$

2. Let the matrix define a game and let Alice be the player whose pure strategies are represented by the rows of the matrix and Bob be the player whose pure strategies are represented by the columns of the matrix. What is the optimal pure strategy for Alice and what is expected payout given that choice? In other words, if Alice must play the same pure strategy in every turn of the game, what pure strategy should she play and what is the expected payout assuming Bob plays optimally? Determine the same information for Bob.

$$\begin{pmatrix} 4 & 8 & 1 & 2 & 6 & 7 \\ 8 & 2 & 3 & 5 & 9 & 2 \\ 1 & 8 & 1 & 7 & 3 & 3 \\ 6 & 5 & 6 & 8 & 6 & 3 \\ 2 & 6 & 2 & 4 & 7 & 1 \\ 3 & 9 & 1 & 5 & 2 & 6 \end{pmatrix}.$$

3. Solve the game with the payoff matrix

$$\begin{pmatrix} 3 & 5 & 3 & -2 & 0 \\ 3 & 7 & 3 & -1 & 1 \\ 2 & -4 & 1 & 3 & 4 \\ 1 & -5 & 1 & 3 & 0 \end{pmatrix}$$

Here both players are allowed to use mixed strategies. You must use dominance relation to reduce the matrix to a 2×3 matrix. You can either use the simplex method or a linear program solver to get a optimal mixed strategy for the row player Alice, but you should express the answer exactly, i.e. as a fraction. Your answer should be with respect to the original 4×5 matrix.

4. Using the 3×2 matrix you obtained using dominance relations in problem 3, write the linear program that you would solve to get an optimal strategy for the column player Bob, then use your answer to the previous problem and complementary slackness to get an optimal solution for Bob. The final answer should be with respect to the original 4×5 matrix. *You can check your answer with a computer, but you should show your work for this problem.*

5. Recall the game Morra discussed in class: Each player plays either 1 or 2 and then, at the same time, they guess what the other player has played. If exactly one of the two players guesses correctly, then that player wins the *total* amount played. For example, suppose Alice plays 2 and Bob plays 1. If Alices guesses that Bob played 1 and Bob guesses that Alice played 2, then no money changes hands, and if Alices guesses that Bob played 2 and Bob guesses that Alice played 2, then Bob when \$3 from Alice.

   Consider the following modification to the game, suppose that, after hiding, Bob always guesses first, so Alice always knows what Bob has quessed before she guesses. This changes the game as Alice has 4 new additional pure strategies: she can play 1 and repeat Bob's guess, play 1 and guess differently than Bob, play 2 and repeat Bob's guess, or play 2 and guess differently than Bob. So Alice has 8 pure strategies in this game, while Bob still only has 4 pure strategies. Find the matrix that corresponds to this modified game and find a worst case optimal mixed strategy for Alice and find the value of the game.

   You can and should use a computer to solve this linear program, but please write the linear program you are solving clearly and write your answer exactly, i.e. as a fraction