Math 482 **HW1** **8/28/2015** Name: ______________________________

Due Friday, September 4, 2015

Students in section D13 (three credit hours) should solve any four of the following five problems. Students in section D14 (four credit hours) must solve all five problems.

1. A company produces three types of chemicals: chemical A, chemical B and chemical C. They sell chemical A for \$30 per barrel, chemical B for \$20 per barrel, and chemical C for \$10 per barrel. Chemical A requires .2 units of energy and 5 units of raw material to produce, Chemical B requires .3 units of energy and 6 units of raw material to produce and Chemical C requires .2 units of energy and 3 units of raw material to produce. Assume that all of the chemicals the company produces are sold. The company must produce at least 55 barrels of chemicals per day. It can use at most 7 units of energy per day and at most 120 units of raw material each day. The company wishes to maximize its profits. Formulate the appropriate linear program in standard form.

2. Solve the following problem, i.e. if the following program has a optimal solution you must find an optimal solution and compute the value of the objective function at the optimal solution, otherwise you must state whether the program is infeasible or unbounded. You should draw the feasible region in the plane.

$$z = 3x_1 + 2x_2 \quad \longrightarrow \quad \max$$

with respect to

$$\begin{cases} x_1 & +4x_2 & \leq & 12, \\ x_1 & +x_2 & \leq & 4, \\ 5x_1 & +2x_2 & \leq & 15, \\ & x_1, x_2 & \geq & 0. \end{cases}$$

3. State the following LP in standard form.

$$z = x_1 + 2x_2 - 3x_3 \quad \longrightarrow \quad \min$$

with respect to

$$\begin{cases} 4x_1 & +x_2 & +2x_3 & & \leq & 3, \\ x_1 & -x_2 & -4x_3 & & \geq & -7, \\ 2x_1 & +3x_2 & & -x_4 & = & 5, \\ x_1, & & x_3 & x_4 & \geq & 0. \end{cases}$$

4. Solve by finding all basic feasible solutions.

$$z = 4x_1 + 2x_2 \quad \longrightarrow \quad \min$$

subject to

$$\begin{cases} 2x_1 + 6x_2 & = 10 \\ x_1, x_2 & \geq 0. \end{cases}$$

5. We are given two instances of LP in standard form:

$$\text{Minimize } z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

subject to
$$\begin{cases} \mathbf{Ax} & = & \mathbf{b}, \\ \mathbf{x} & \geq & \mathbf{0} \end{cases}$$

and

$$\text{Minimize } \xi = -c_1x_1 - c_2x_2 - \ldots - c_nx_n$$

subject to
$$\begin{cases} \mathbf{Ax} & = & \mathbf{b}, \\ \mathbf{x} & \geq & \mathbf{0}. \end{cases}$$

The numbers $c_1, \ldots, c_n$, matrix $\mathbf{A}$, and vector $\mathbf{b}$ are the same in both instances. Does there exists $\mathbf{A}$, $\mathbf{b}$ and $c_1 \ldots, c_n$ such that both instances have feasible solutions with arbitrarily small cost, i.e. for any $M \in \mathbb{R}$ does there exists $\mathbf{x}, \mathbf{x}' \geq 0$ such that $\mathbf{Ax} = \mathbf{Ax}' = b$, $c_1x_1 + c_2x_2 + \ldots + c_nx_n < M$ and $-c_1x_1' - c_2x_2' - \ldots - c_nx_n' < M$.

If 'yes', give an example; if 'not', prove so.