

## Final topics

Only the D14 students need to know the proofs of the theorem marked with “(with proof)”. Students in both D13 and D14 sections are required to know and understand the statement of the theorems listed. The best way to prepare for the final is to review old tests, homework and quizzes. You should also know the definitions and statement of theorems listed below. The theorems and proofs can be found on the website or in the book.

- (1) branch and bound - steps of algorithm branch and bound algorithm for integer programs conditions when one node kills another node
- (2) (Koenig’s theorem) Min vertex cover = max matching in bipartite graphs - via integer programming
- (3) Integer programming and satisfiability
- (4) Theorem 13.3 (with proof)
- (5) The incidence matrix of bipartite graph and directed graphs are TUM (Theorem 13.3 Corollary)
- (6) Relaxation of a integer linear program
- (7) The basic feasible solutions the LP in standard or canonical form are integer if the matrix is totally unimodular
- (8) Totally unimodular matrices - definition
- (9) Max-flow = min-cut
- (10) How to find a min-cut in a flow network
- (11) Min-cost flow
- (12) Algorithm cycle for min-cost flow
- (13) incremental weighted flow network  $N'(f)$
- (14) Floyd-Warshall algorithm including reconstructing a path with the  $E^k$  matrix
- (15) Two phase revised simplex
- (16) Max-flow via revised simplex (Section 4.3)
- (17) Value of flow  $f$ :  $|f|$
- (18) Arc-Chain incidence matrix
- (19) Primal dual algorithm
- (20) Primal (P), Dual (D), Restricted Primal (RP) and dual of restricted primal (DRP)
- (21) Admissible columns
- (22) Computing  $\theta$
- (23) Theorem 5.1 - with proof
- (24) Theorem 5.3 - with proof
- (25) Dijkstra’s Algorithm (section 6.4)
- (26) Primal dual for shortest path
- (27) Ford-Fulkerson algorithm for max-flow
- (28)  $f$ -augmenting  $(s, t)$ -path
- (29) Weak Duality (with proof) -  $\pi^T b \leq c^T x$  (see the first part of Theorem 3.1 - you can assume the LPs are in canonical form).
- (30) Finding the dual of a linear program in general form (Definition 3.1)
- (31) Strong Duality (Theorem 3.1)
- (32) dual simplex method (Section 3.6)
- (33) Farkas Lemma (Theorem 3.5)
- (34) Matrix games, pure strategy, mixed strategy, minimax theorem, stochastic vertex, Alice, Bob, optimal strategy, value of game, solving a matrix game using linear programming (see handout on website)
- (35) complementary slackness (section 3.4) (with proof)
- (36) incidence matrices of digraphs
- (37) shortest path problem as an LP (Section 3.4),
- (38) A circulation is the sum of flows on cycles, and a  $(s, t)$ -flow is the sum of flows on  $(s, t)$ -path, cycles and  $(t, s)$ -paths

- (39) revised simplex method (section 4.1)
- (40) Definition: feasible solution
- (41) Definition: object function
- (42) Definition: optimal solution, optimum
- (43) Definition: basic feasible solution
- (44) Definition: degenerate basic feasible solution
- (45) Definition: relative cost of column  $j$
- (46) Definition: Standard/Canonical/General form
- (47) Definition: lex positive, lex negative, lex zero
- (48) Converting between forms of linear program - slack variables and surplus variables
- (49) Solving a 2d LP graphically
- (50) Simplex method and two phase simplex method (see examples on website)
- (51) Lexicographic simplex - initialization of row to be lex positive and pivot rules
- (52) Bland's rule
- (53) Fact that lexicographic simplex and Bland's rule do not cycle
- (54) Proposition about the relationship between a feasible basis and a basic feasible solution (Proposition 2) (with proof)
- (55) Useful characterization of basic feasible solutions (Proposition 4) (with proof)
- (56) Lemma about the existence of a basic feasible solutions (Lemma 5)
- (57) Fundamental theorem (Theorem 6)
- (58) Proposition about the significance of the relative cost vector  $\bar{\mathbf{c}}^T$  (Proposition 7) (with proof)
- (59) Theorem about the pre-multiplication matrix (Theorem 8)