Cycling example (2.7 from book)

- This is an example 2.7 from the book and is an example of cycling.
- The first tableau T_1 will appear again as tableau T_7 when we use the following natural pivot rules.
- Select the pivot column j_{in} so that $\overline{c}_{j_{in}} \leq \overline{c}_j$ for all $j \in [n]$ (In this example, this always gives a unique choice)
- In the case of ties when selecting the pivot row, select the row so that the smallest index leaves the basis (this rule is the same as Bland's rule)

Tableau 1

$$T_1 = \begin{array}{c|c|ccccccc} & & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_5 & 0 & \mathbf{1/4} & -8 & -1 & 9 & 1 & 0 & 0 \\ x_6 & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ x_7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}$$

bfs $x = (0, 0, 0, 0, 0, 1)$, basis $B = (5, 6, 7)$.

Tableau 2

$$T_2 = \begin{array}{c|c|ccccccc} & & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & 0 & -4 & -\frac{7}{2} & 33 & 3 & 0 & 0 \\ \hline x_1 & 0 & 1 & -32 & -4 & 36 & 4 & 0 & 0 \\ x_6 & 0 & 0 & \mathbf{4} & \frac{3}{2} & -15 & -2 & 1 & 0 \\ x_7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}$$

bfs $x = (0, 0, 0, 0, 0, 1)$, basis $B = (1, 6, 7)$.

Tableau 3

$$
T_3 = \begin{array}{c|c|ccccccc}
 & & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\
\hline
-z & 3 & 0 & 0 & -2 & 18 & 1 & 1 & 0 \\
\hline
x_1 & 0 & 1 & 0 & \mathbf{8} & -84 & -12 & 8 & 0 \\
x_2 & 0 & 0 & 1 & \frac{3}{8} & -\frac{15}{4} & -\frac{1}{2} & \frac{1}{4} & 0 \\
x_7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1
\end{array}
$$

bfs $x = (0, 0, 0, 0, 0, 1)$, basis $B = (1, 2, 7)$.

Tableau 4

$$
T_4 = \begin{array}{c|c|ccccccc}
 & & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\
\hline
-z & 3 & \frac{1}{4} & 0 & 0 & -3 & -2 & 3 & 0 \\
\hline
x_3 & 0 & \frac{1}{8} & 0 & 1 & -\frac{21}{2} & -\frac{3}{2} & 1 & 0 \\
x_2 & 0 & -\frac{3}{64} & 1 & 0 & \mathbf{\frac{3}{16}} & \frac{1}{16} & -\frac{1}{8} & 0 \\
x_7 & 1 & -\frac{1}{8} & 0 & 0 & 21/2 & \frac{3}{2} & -1 & 1
\end{array}
$$

bfs $x = (0, 0, 0, 0, 0, 1)$, basis $B = (3, 2, 7)$.

Tableau 5

$$
T_5 = \begin{array}{c|c|ccccccc}
 & & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\
\hline
-z & 3 & -\frac{1}{2} & 16 & 0 & 0 & -1 & 1 & 0 \\
\hline
x_3 & 0 & -\frac{5}{2} & 56 & 1 & 0 & \mathbf{2} & -6 & 0 \\
x_4 & 0 & -\frac{1}{4} & \frac{16}{3} & 0 & 1 & \frac{1}{3} & -\frac{2}{3} & 0 \\
x_7 & 1 & \frac{5}{2} & -56 & 0 & 0 & -2 & 6 & 1
\end{array}
$$

bfs $x = (0, 0, 0, 0, 0, 1)$, basis $B = (3, 4, 7)$.

Tableau 6

$$T_6 = \begin{array}{c|c|ccccccc} & & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & -\frac{7}{4} & 44 & \frac{1}{2} & 0 & 0 & -2 & 0 \\ \hline x_5 & 0 & -\frac{5}{4} & 28 & \frac{1}{2} & 0 & 1 & -3 & 0 \\ x_4 & 0 & \frac{1}{6} & -4 & -\frac{1}{6} & 1 & 0 & \mathbf{\frac{1}{3}} & 0 \\ x_7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}$$

bfs $x = (0, 0, 0, 0, 0, 1)$, basis $B = (5, 4, 7)$.

Tableau 7 same as Tableau 1!

$$T_7 = \begin{array}{c|c|ccccccc} & & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline -z & 3 & -\frac{3}{4} & 20 & -\frac{1}{2} & 6 & 0 & 0 & 0 \\ \hline x_5 & 0 & \frac{1}{4} & -8 & -1 & 9 & 1 & 0 & 0 \\ x_6 & 0 & \frac{1}{2} & -12 & -\frac{1}{2} & 3 & 0 & 1 & 0 \\ x_7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}$$

bfs $x = (0, 0, 0, 0, 0, 1)$, basis $B = (5, 6, 7)$.

Example with Lexicographic simplex

- We start with the same tableau as example 2.7 from the book, but this time we following the lexicographic simplex method.
- That is, in the case of ties when selecting the pivot row, we select the row that is smallest in lexicographic order.

Tableau 1

$T_1 =$

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
$-z$	3	$-\frac{3}{4}$	20	$-\frac{1}{2}$	6	0	0	0
x_5	0	$\mathbf{1/4}$	-8	-1	9	1	0	0
x_6	0	$\frac{1}{2}$	-12	$-\frac{1}{2}$	3	0	1	0
x_7	1	0	0	1	0	0	0	1

bfs $x = (0,0,0,0,0,1)^T$, basis $B = (5,6,7)$.

Rows 1 and 2 are tied. But row 1 divided by $a_{1,1}$ is lexicographically less that row 2 divied by $a_{2,1}$ ($a_{1,3}/a_{1,1} = -32 < -24 = a_{2,3}/a_{2,1}$)

Tableau 2

$T_2 =$

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
$-z$	3	0	-4	$-\frac{7}{2}$	33	3	0	0
x_1	0	1	-32	-4	36	4	0	0
x_6	0	0	$\mathbf{4}$	$\frac{3}{2}$	-15	-2	1	0
x_7	1	0	0	1	0	0	0	1

bfs $x = (0,0,0,0,0,1)^T$, basis $B = (1,6,7)$.

There is no choice for the selection of the pivot row. This step is also the same as the example in the book.

Tableau 3

$T_3 =$

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
$-z$	3	0	0	-2	18	1	1	0
x_1	0	1	0	8	-84	-12	8	0
x_2	0	0	1	$\mathbf{\frac{3}{8}}$	$-\frac{15}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	0
x_7	1	0	0	1	0	0	0	1

bfs $x = (0,0,0,0,0,1)^T$, basis $B = (1,2,7)$.

In this step, rows 1 and 2 are tied, and we select row 2 because it is lexicographically less than row 1, ($a_{1,0}/a_{1,3} = a_{2,0}/a_{2,3} = 0$, and $a_{2,1}/a_{2,3} = 0 < 1/8 = a_{1,1}/a_{1,3}$). this step is different than the example in the book.

Note that if we were following Bland's rule, we would pick row one instead of row two. This is because row 1 is solved for x_1 and row 2 is solved for x_2 and x_1 has a lower index than x_2.

Tableau 4

$$
T_4 = \begin{array}{r|c|ccccccc|}
 & & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\
\hline
-z & 3 & 0 & 16/3 & 0 & -2 & -5/3 & 7/3 & 0 \\
\hline
x_1 & 0 & 1 & -64/3 & 0 & -4 & -4/3 & 8/3 & 0 \\
x_3 & 0 & 0 & 8/3 & 1 & -10 & -4/3 & 2/3 & 0 \\
x_7 & 1 & 0 & -8/3 & 0 & 10 & \mathbf{4/3} & -2/3 & 1
\end{array}
$$

bfs $x = (0, 0, 0, 0, 0, 1)^T$, basis $B = (3, 2, 7)$.

We pick column 5 as our pivot column, because we happen to realize that that choice will make this the final pivot, but x_4 would also be a valid choice. Once we select column 5 as our pivot column, we must pivot on row 3

Tableau 5

$$
T_5 = \begin{array}{r|c|ccccccc|}
 & & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\
\hline
-z & 17/4 & 0 & 2 & 0 & 21/2 & 0 & 3/2 & 5/4 \\
\hline
x_1 & 1 & 1 & -24 & 0 & 6 & 0 & 2 & 1 \\
x_3 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
x_5 & 3/4 & 0 & -2 & 0 & 15/2 & 1 & -1/2 & 3/4
\end{array}
$$

bfs $x = (1, 0, 1, 0, 3/4, 0)^T$, basis $B = (1, 3, 5)$.

This is the optimal solution, because the entries in the top row (columns 1 thru 7) are non-negative.

Quiz

- Find the pivot entry using Bland's rule and lexicographic simplex

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
$-z$	8	2	0	0	2	-3	0	0
x_7	2	2	0	0	3	4	0	1
x_6	6	10	0	0	4	12	1	0
x_3	4	5	0	1	2	8	0	0
x_2	2	0	1	0	4	3	0	0

- Using Bland's rule we select column x_5 and row 3
- Using lexicographic simplex we select column x_5 and row 1