Math-484 Homework #9 (penalty method)
Due 11am Nov 19.

1: Consider the following program:

$$(P)\begin{cases}\text{Minimize} & f(x) = x^2 - 2x\\ \text{subject to} & 0 \le x \le 1.\end{cases}$$

(a) Sketch the graphs of the Absolute Value and Courant-Beltrami Penalty Terms for (P).
(b) For each positive integer k, compute the minimizer x_k of the corresponding unconstrained objective function $P_k(x)$ with the Courant-Beltrami Penalty Term.
(c) For each positive integer k, compute the minimizer x_k of the corresponding unconstrained objective function $F_k(x)$ with the Absolute Value Penalty Term.

2:

a) Use the penalty function method with the Courant-Beltrami penalty term to solve the problem (P).

$$(P)\begin{cases}\text{Minimize} & f(x_1, x_2) = x_1 + x_2\\ \text{subject to} & x_1^2 - x_2 \le 2\end{cases}$$

b) Show that the objective function $F_k(\mathbf{x})$ corresponding to the Absolute value penalty term has no critical points off the parabola

$$x_1^2 - x_2 = 2$$

for $k > 1$ and compute the minimizer of $F_k(\mathbf{x})$.

3: Use the Penalty Function Method with Courant-Beltrami Penalty Term to minimize

$$f(x, y) = x^2 + y^2$$

subject to constraint $x + y \ge 1$.

4: Let $\varepsilon > 0$. Show that if a vector λ is a feasible for the dual (D) of a convex program (P), then λ is also feasible for (D^ε).

5: We know that if a convex program is superconsistent, then $MP = MD$. Show that the converse is not true. That is: find a convex program that is not sueprconsistent and yet $MP = MD$.
Hint: You can use the fact the for all linear programs $MP = MD$.

6: *(D14 only)* Let $g(x)$ be a differentiable fuction on $\mathbb{R}^1$ and suppose that $g(x_0) = 0$ for some $x_0 \in \mathbb{R}^1$.

(a) Show that $g^+(x)$ is differentiable at x_0 if and only if $g'(x_0) = 0$.

(b) Show carefully that $(g^+(x))^2$ is differentiable at x_0 and that its derivative at x_0 is zero.