Math-484 Homework #8 (constrained geometric programs, linear programs, quadratic programs)

Due 11am Nov 12.
Write your name on your solutions and indicate if you are a D14 (4 credit hour) student.

1: Reduce the following problem to a standard geometric programming problem and then solve it using the geometric programming procedure discussed in class.

$$(P)\begin{cases}\text{Minimize} & \sqrt{3}(x^2+y^2)^{1/2}+\dfrac{2}{x^2y}\\ \text{subject to} & x>0, y>0\end{cases}$$

You should solve this by hand, but you should check your solution using `http://www.wolframalpha.com` or a similar system.

2: You have \$12,000 to invest, and three different funds from which to choose. The municipal bond fund has a 7% return, the local bank's CDs have an 8% return, and the high-risk account has an expected (hoped-for) 12% return. To minimize risk, you decide not to invest any more than \$2,000 in the high-risk account. For tax reasons, you need to invest at least three times as much in the municipal bonds as in the bank CDs. Assuming the year-end yields are as expected, what are the optimal investment amounts?

Formulate the question using linear programming using two variables and solve the program by examining the set of the feasible solutions (plot the set of feasible points in 2D). Invest all money you have.

3: Write the dual to the following program and then solve the dual by plotting the feasible region on the plane. Then use complementary slackness and the duality theorem to solve the original linear program.

$$(P)\begin{cases}\text{Minimize} & 3x+4y+2z\\ \text{subject to} & y+2z\geq 10\\ & x+y-z\geq 1\\ & x\geq 0, y\geq 0, z\geq 0\end{cases}$$

4: Solve the following quadratic program,

$$(P)\begin{cases}\text{Minimize} & 2x^2+xy+y^2\\ \text{subject to} & x+3y\leq -8\end{cases}$$

5: Convert the following program to a linear program (i.e. avoid using absolute values). You do not need to solve this program.

$$(P)\begin{cases}\text{Minimize} & \max\{|x|,|y|,|z|\}| \\ \text{subject to} & x+y\leq 0 \\ & 2x+z=3\end{cases}$$

6: Let $f(x)$ be a differentiable function on $\mathbb{R}$. Suppose x_0 is fixed and there exists a number $\alpha\in\mathbb{R}$ such that

$$f(x)\geq f(x_0)+\alpha(x-x_0)$$

for all $x\in\mathbb{R}$. Show that $\alpha=f'(x_0)$.