Math-484 Homework #7 (KKT conditions)

Due 11am Nov 7.
Write your name on your solutions and indicate if you are a D14 (4 credit hour) student.

1: Let (P) be a convex program and let

$$S := \{x \in C : f(x) = MP \text{ and } g_i(x) \leq 0 \text{ for all } i \in [n]\ \}.$$

Prove that S is a convex set.
Hint: First show that if f is convex on C, then for any $m \in \mathbb{R}$, the set $\{x \in C : f(x) \leq m\}$ is convex.

2: Apply the Karush-Kuhn-Tucker Theorem to locate all solutions of the following convex programs:

$$(P_a)\begin{cases}\text{Minimize} & f(x_1,x_2) = e^{-(x_1+x_2)}\\ \text{subject to} & e^{x_1}+e^{x_2} \leq 20\\ & x_1 \geq 0\end{cases} \qquad (P_b)\begin{cases}\text{Minimize} & f(x_1,x_2) = x_1^2+x_2^2-4x_1-4x_2\\ \text{subject to} & x_1^2 - x_2 \leq 0\\ & x_1+x_2 \leq 2\end{cases}$$

3: Apply the Karush-Kuhn-Tucker Theorem to locate all solutions of the following convex program:

$$(P)\begin{cases}\text{Minimize} & -x_1+x_2\\ \text{subject to} & x_1^2+x_1-x_2-2 \leq 0\\ & 11x_1+5x_2-6 \leq 0\end{cases}$$

4: Consider the following geometric program:

$$(GP)\begin{cases}\text{Minimize} & f(t_1,t_2) = t_1^{-1}t_2^{-1}\\ \text{subject to} & \frac{1}{2}t_1+\frac{1}{2}t_2 \leq 1\\ \text{where} & t_1>0,\ t_2>0\end{cases}$$

a) Convert (GP) to an equivalent convex program and solve the resulting program using KKT.
b) Solve the given (GP) by using methods of Chapter 5.3.

5: Solve the following geometric program:

$$(GP)\begin{cases}\text{Minimize} & x^{1/2}+y^{-2}z^{-1}\\ \text{subject to} & x^{-1}y^2+x^{-1}z^2 \leq 1\\ \text{where} & x>0,\ y>0,\ z>0\end{cases}$$

6: *(D14 only)* Let A be an $m \times n$ matrix and let $\mathbf{b} \in \mathbb{R}^m$ be a fixed vector. Suppose that the convex program

$$(P) \begin{cases} \text{Minimize} & \|\mathbf{x}\|^2 \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b} \end{cases}$$

is superconsistent and has a solution $\mathbf{x}^*$. Use Karush-Kuhn-Tucker Theorem to show that there is a vector $\mathbf{y} \in \mathbb{R}^m$ such that $\mathbf{x}^* = A^T\mathbf{y}$.