Math-484 Homework #6 (least squares, convex sets)

Due 11am Oct 22.
Write your name on your solutions and indicate if you are a D14 (4 credit hour) student.

1: Compute the best least square fit for polynomial

$$p(t) = x_0 + x_1 t + x_2 t^2$$

and data

t_i	-2	-1	0	1	2	3	4
s_i	-5	-1	4	7	6	5	-1

2: Find best least squares solution to inconsistent linear system using QR factorization.

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 4 & 6 \\ 1 & 1 & 0 \\ 1 & 4 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 2 \\ 2 \end{pmatrix}$$

3: (a) Find the point on the plane

$$x + 2y + z = 6$$

that is closest to the origin.
(b) Find the minimum norm solution of the underdetermined linear system

$$\begin{aligned} 2x_1 + x_2 + x_3 + 5x_4 &= 8 \\ -x_1 - x_2 + 3x_3 + 2x_4 &= 0 \end{aligned}$$

4: Find vector $\mathbf{x} \in \mathbb{R}^3$ that is closest to $(1,1,1)$ where $\alpha, \beta \in \mathbb{R}$ and

$$\mathbf{x} = \alpha(1,1,2) + \beta(2,-1,1)$$

5: Let C be a closest convex subset of $\mathbb{R}^n$. If $\mathbf{y} \notin C$, show that $\mathbf{x}^* \in C$ is the closest vector to $\mathbf{y}$ in C if and only if $(\mathbf{x} - \mathbf{y})^T(\mathbf{x}^* - \mathbf{y}) \geq ||\mathbf{x}^* - \mathbf{y}||^2$ for all $\mathbf{x} \in C$.

6: Prove that if M is a subspace of $\mathbb{R}^n$ such that $M \neq \mathbb{R}^n$, then the interior M^0 of M is empty.
Hint: Use that the orthogonal complement $M^\perp$ of M is also a subspace. Recall $M^\perp = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{x}^T\mathbf{y} = 0$ for all $y \in M\}$. You should use the fact that $M^\perp \cap M = \{0\}$.