Math-484 Homework #5 ($(A-G)$ inequality, least squares, open and closed sets)

Due 11am Oct 15.
Write your name on your solutions and indicate if you are a D14 (4 credit hour) student.

1: State the dual (DGP) of the following (GP) and solve the (GP) using (DGP). Solving means, finding optimal $\mathbf{x}^* = (x_1, x_2)$ and value of the objective function.

$$(GP)\begin{cases}\text{Minimize} & (5^4)\frac{x_2^2}{x_1} + \frac{x_3}{5x_1x_2^2} + \frac{25x_1}{2} + \frac{1}{10x_1x_3^2} \\ \text{subject to} & x_1, x_2, x_3 > 0\end{cases}$$

You should verify your answers using `http://www.wolframalpha.com` or equivalent.

2: Find the least squares solution of the inconsistent linear system of 6 equations:

$$\begin{aligned} x_1 + x_2 + x_3 &= 3 & \qquad x_3 &= 1 \\ x_1 + x_3 &= 2 & \qquad 2x_1 + 5x_3 &= 8 \\ -7x_1 + 8x_2 &= 0 & \qquad x_1 + 2x_2 - x_3 &= 1 \end{aligned}$$

You should compute the matrix equation $A^TAx = A^Tb$ by hand and then use a software system to find x^*, the best least square solution to the system (an approximate answer is OK). Compute the error your solution gives in each of the 6 equations.

3: Compute the equation of the linear regression line corresponding to the data on the table below:

x	-2	-1	0	1	2	3
y	12	11	8	5	2	-3

4: Compute generalized inverse $A^\dagger$ of

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 3 \end{pmatrix}$$

5: Find orthonormal bases of linear subspaces generated by:
a) $L_1 = \{(0,3,4,0)^T, (0,0,5,0)^T, (2,1,0,2)^T\}$
b) $L_2 = \{(2,0,1,2)^T, (4,3,2,4)^T, (6,-5,3,6)^T, (-4,2,4,2)^T\}$

6: *(D14 only)* Let $\mathcal{F}$ be the set of all functions $f : \mathbb{R} \to \mathbb{R}$ where $f(x) > 0$ for every $x \in \mathbb{R}$. Let

$$\|f - g\| = \sup_{x \in \mathbb{R}} |f(x) - g(x)|.$$

Determine if $\mathcal{F}$ is convex, open and/or closed.