Math-484 Homework #1

Due 11am Sep 3. Write your name and indicate if you are a C14 (4 credit hour) student.

1: (*Minimizers and maximizers of smooth functions*)
Find the local and global minimizers and maximizers of the following functions:
(a) $f(x) = x^2 + 2x$
(b) $f(x) = x^2 e^{-x^2}$
Breifly explain why your answers are correct. You can verify your answers using `http://www.wolframalpha.com` or equivalent.
Hint: Use the first and second derivatives of $f(x)$ and critical points.

2: (*Techniques from linear algebra*)
Determine the dimension of the smallest subspace of $\mathbb{R}^4$ that contains vectors $(0, 1, 0, 1)$, $(3, 4, 1, 2)$, $(6, 4, 2, 0)$ and $(-3, 1, -1, 3)$.
Hint: You should construct a matrix and use Gaussian elimination.

3: (*Practice with determinants*)
Compute the determinants of the following real matrices:

$$\text{(a)} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \qquad \text{(b)} \begin{pmatrix} 0 & -2 & 1 & 0 \\ 4 & a & b & 1 \\ 1 & c & d & 4 \\ 0 & 1 & -2 & 0 \end{pmatrix} \text{ where } a, b, c, d \in \mathbb{R} \text{ are parameters}$$

You can verify your answers using `http://www.wolframalpha.com` or equivalent.
Hint: For (b) expand by cofactors, and note that the parameters do not necessarily have to appear in the answer.

4: (*Recall how to compute eigenvalues and eigenvectors*)
Compute the eigenvalues and eigenvectors of the following real matrix

$$A = \begin{pmatrix} 2 & 6 \\ 6 & -3 \end{pmatrix}$$

5: (*Compute the angle between two vectors in $\mathbb{R}^n$ when $n > 2$*)
Compute the angle between the vectors $(1, 1, 0)$ and $(2, 2, \sqrt{2})$.

6: (*Semidefinitness, norms and practice multiplying matrices and vectors.*)
Suppose that A is a square matrix and suppose that there is another matrix B such that $A = B^T B$. Prove that A is positive semidefinite.
Hint: Use the definition of a positive semidefinite matrix, and the fact that $\mathbf{y}^T B^T \mathbf{x} = (B\mathbf{y})^T \mathbf{x}$ This proof should be short.