Math-484 List of definitions and theorems

The makeup exam (Midterm 4) will cover the same material as Midterm 3, except there will be a one or two questions covering material from Midterms 1 and 2. The following is a short list of some topics from the older material that you should make sure to review if you are taking the make-up exam on December 9th. (*Proofs are only for D14 (4 credit hour) students*)

- Taylor's formula for functions from $\mathbb{R}^n$ to $\mathbb{R}$ *(Theorem 1.2.4)*
- Coercive functions and minimization *(Theorem 1.4.4, **with proof**)*
- Building convex function from other convex functions *(Theorem 2.3.10)*
- Principal minors of matrix A and there relation to positive(negative) (semi)definite matrices A *(Theorem 1.3.3)*
- Eigenvalues of a symmetric matrix and there relation to positive(negative) (semi)definite matrices *(Theorem 1.5.1)*
- Describe transition form unconstrained geometric program to its dual using A-G inequality (pages 67, 68).
- How to compute a best least squares solution of a system $A\mathbf{x} = \mathbf{b}$? *Theorem 4.1.2*
- How to compute P_M (orthogonal projection of $\mathbb{R}^m$ onto M) and the properties of P_M? *Theorem 4.2.5*

Definitions (Midterm 3):

- epi(f) *page 167*
- feasible point (or feasible vector) of a program (P) *page 169*
- feasible region of a program (P) *page 169*
- consistent program (P) *page 169*
- Slater point of a program (P) *page 169*
- superconsistent program (P) *page 169*
- solution of a program (P) *page 169*
- convex program (P) *page 169*
- MP for program (P) *page 172*
- linear program (LP) *page 173*
- Perturbation of P by z, $P(z)$ *page 174*
- $MP(z)$ *page 174*
- sensitivity vector of a program (P) *page 177*
- Lagrangian $L(\mathbf{x}, \lambda)$ of a program (P)*page 182*
- complementary slackness conditions for a program (P) *page 184*
- saddle point of the Lagranian of (P) *page 184*
- KKT multipliers *page 187*
- general form of constrained geometric program (GP) and its dual (DGP) *page 193*
- dual of a convex program *page 200-201*
- $h(\lambda)$ *page 201*
- MD *page 201*
- feasible vector for the dual (DP) *page 201*
- consistent dual program (DP) *page 201*
- solution of the dual program (DP) *page 201*
- dual of a linear program (DLP) *page 202*
- duality gap *page 209*
- absolute value penalty function *page 217*
- Courant-Beltrami penalty function *page 219*
- generalized penalty function *page 223*
- $f^\varepsilon(x)$ where $f(x)$ is a convex function *page 229*
- (P^ε), (DP^ε), MP^ε, MD^ε, $h^\varepsilon(x)$, $L^\varepsilon(x, \lambda)$ when $\varepsilon > 0$ and (P) is a convex program *page 230*

Theorems and statements (Midterm 3):

(*Try to not ignore assumptions - like if a function must be continuous etc.*)

(*Proofs are only for D14 (4 credit hour) students*)

- What is a sufficient condition for existence of a unique closest vector from a set C to a given vector $\mathbf{x}$? *Corollary 5.1.4*
- State basic separation theorem. *Theorem 5.1.5*
- State Support theorem. *Theorem 5.1.9*
- State *Theorem 5.1.10*
- What can you say about the function $MP(z)$ when (P) is superconsistent? *Theorem 5.2.6*
- Are there sufficient conditions for convex program (P) to have a sensitivity vector? *Theorem 5.2.8,* ***with proof***
- Can MP be computed from the sensitivity vector? *(Theorem 5.2.11),* ***with proof***
- State Karush-Kuhn-Tucker Theorem (Saddle point version) *Theorem 5.2.13*
- State Karush-Kuhn-Tucker Theorem (Gradient form) *Theorem 5.2.14*
- State Extended Arithmetic-Geometric Mean Inequality include the condition for equality! *Theorem 5.3.1,* ***with proof***
- What are sufficient condition for a constrained geometric program (GP) to have no duality gap? *Theorem 5.3.5*
- State the (strong) duality theorem for linear programming *page 203*
- State the duality theorem for convex programming. *Theorem 5.4.6*
- State the theorem that gives properties of Courant-Beltrami penalty function. *Theorem 6.2.3*
- State the theorem about the convergence of the sequence of minimizers of the Courant-Beltrami penalty functions when the objective function is coercive. *Theorem 6.2.4* ***with proof***
- What is the effect of the coercive objective function on the duality? *Theorem 6.3.1*
- What can you say when (P) is superconsistent and $MP > -\infty$? *Theorem 6.3.5*?

Definitions (Midterm 2, these may appear on Midterm 3):
- posynomial *page 67*
- unconstrained geometric program
- primal and dual geometric program *page 67,68*
- feasible solution
- consistent program
- best least squares kth degree polynomial *page 135*
- linear regression line *page 135*
- best least squares solution of linear system $A\mathbf{x} = \mathbf{b}$ *page 136*
- generalized inverse of a matrix $A \in \mathbb{R}^{m\times n}$ *page 136*
- orthonormal vectors *page 138*
- QR of a matrix (page 139) - subspace of $\mathbb{R}^n$ *page 141*
- orthogonal complement of a subspace of $\mathbb{R}^n$ *page 142*
- P_M - orthogonal projection of R^m onto M *page 144*
- underdetermined system of linear equations *page 145*
- H-inner product *page 149*
- H-norm *page 149*
- H-orthogonal vectors *page 149*
- H-orthogonal complement *page 149*
- H-generalized inverse*page 150*
- hyperplane H in $\mathbb{R}^n$ *page 158*
- boundary point of $C \subset \mathbb{R}^n$ *page 158*
- closure $\overline{A}$ of $A \subset \mathbb{R}^n$ *page 163*

Theorems and statements (Midterm 2):
(*Try to not ignore assumptions - like if a function must be continuous etc.*)
(*Proofs are only for D14 (4 credit hour) students*)
- Describe transition form unconstrained geometric program to its dual using A-G inequality (pages 67, 68).
- How to compute a best least squares solution of a system $A\mathbf{x} = \mathbf{b}$? *Theorem 4.1.2*
- What is the QR factorization of a matrix A and when does it exists? *Theorem 4.1.5*
- How to compute P_M? (orthogonal projection of $\mathbb{R}^m$ onto M) *Theorem 4.2.5*
- If $M \subseteq \mathbb{R}^m$ is a subspace, then what is $\left(M^\perp\right)^\perp$? *(Theorem 4.2.7)*
- What is the form of solutions of underdetermined systems? *Theorem 4.3.1,* ***with proof***
- What is the form of minimum norm solutions of underdetermined systems? *(Theorem 4.3.2,* ***with proof****)*
- What is the form of minimum H-norm solutions of underdetermined systems? *(Theorem 4.4.2)*
- If $C \subseteq \mathbb{R}^n$ is a convex set $y \in \mathbb{R}^n \setminus C$, then what is true if and only if $x^* \in C$ is the closest vector to y in C? *(Theorem 5.1.1)*
- What is the characterization of the closest vector of a convex set to a given vector using orthogonal complement? *(Theorem 5.1.2,* ***with proof****)*

- What is a sufficient condition for existence of a closest vector from a set C to a given vector $\mathbf{x}$? *(Theorem 5.1.3)*

Definitions (from Midterm 1, these may appear on Midterm 3):

- cosine of two vectors *page 6*
- distance of two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ *page 6*
- ball $B(\mathbf{x}, r)$ (what is $\mathbf{x}$ and r?) *page 6*
- interior D^0 of set $D \subseteq R^n$ *page 6, page 164*
- open set $D \subseteq \mathbb{R}^n$ *page 6*
- closed set $D \subseteq \mathbb{R}^n$ *page 7*
- compact set $D \subseteq \mathbb{R}^n$ *page 6*
- (global,local)(strict)minimizer and maximizer of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 8*
- critical point of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 8*
- gradient $\nabla f(\mathbf{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ *page 10*
- Hessian $Hf(\mathbf{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ *page 10*
- quadratic form associated with a symmetric matrix A *page 12*
- (positive,negative)(semi)definite matrix *page 13*
- indefinite matrix *page 13*
- saddle point of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 23*
- Δ_k, the k^{th} principal minor of a matrix A *page 16*
- coercive functions *page 25*
- eigenvalues and eigenvectors of a matrix A *page 29*
- convex sets in $\mathbb{R}^n$ *page 38*
- closed and open half-spaces in $\mathbb{R}^n$ *page 40*
- convex combination of k vectors from $\mathbb{R}^n$ *page 41*
- convex hull of $D \subseteq \mathbb{R}^n$ *page 42*
- (strictly) convex and concave function $f : C \to \mathbb{R}$, where $C \subseteq \mathbb{R}^n$ *page 49*

Theorems and statements (from Midterm 1):

(*Try to not ignore assumptions - like if a function must be continuous etc.*)
(*Proofs are only for D14 (4 credit hour) students*)

- State Cauchy-Swartz inequality *(page 6)*
- Minimizers and maximizers of continuous function $f : I \to \mathbb{R}$ where $I \subset \mathbb{R}$ is a closed interval *(Theorem 1.1.4)*
- local minimizers and the gradient *(Theorem 1.2.3)*
- Taylor's formula for $\mathbb{R}^n$ *(Theorem 1.2.4)*
- H_f and global minimizers and maximizers? *(Theorem 1.2.5/Theorem 1.2.9)*
- Principal minors of matrix A and there relation to positive(negative) (semi)definite matrices A *(Theorem 1.3.3)*
- Eigenvalues of a symmetric matrix and there relation to positive(negative) (semi)definite matrices *(Theorem 1.5.1)*
- H_f and local minimizers and maximizers. *(Theorem 1.3.6, **with proof**)*

- coercive functions and minimization *(Theorem 1.4.4, **with proof**)*
- The convex hull of $D \subseteq \mathbb{R}^n$, $co(D)$, is the set of all convex combinations of vectors from D. ($D \subseteq \mathbb{R}^n$) *(Theorem 2.1.4)*
- convex function and continuity *(Theorem 2.3.1)*
- minimizers of convex functions *(Theorem 2.3.4 **with proof**)*
- maximizers of concave functions *(Theorem 2.3.4)*
- the relationship between convex function and the gradient *(Theorem 2.3.5)*
- critical points of convex function and minimization *(Theorem 2.3.5 + Corollary 2.3.6)*
- The relationship between the Hessian and convexity of a function (in $\mathbb{R}^n$) *(Theorem 2.3.7)*
- building convex function from other convex functions *(Theorem 2.3.10)*
- inequality involving convex functions and convex combinations with the condition for equality (finite version of Jensen's Inequality) *(Theorem 2.3.3)*
- arithmetic-geometric mean inequality with the condition for equality *(Theorem 2.4.1 **with proof**)*