Math-484 List of definitions and theorems

Definitions (Midterm 2):

- posynomial *page 67*
- unconstrained geometric program
- primal and dual geometric program *page 67,68*
- feasible solution
- consistent program
- best least squares kth degree polynomial *page 135*
- linear regression line *page 135*
- best least squares solution of linear system $A\mathbf{x} = \mathbf{b}$ *page 136*
- generalized inverse of a matrix $A \in \mathbb{R}^{m\times n}$ *page 136*
- orthonormal vectors *page 138*
- QR of a matrix (page 139) - subspace of $\mathbb{R}^n$ *page 141*
- orthogonal complement of a subspace of $\mathbb{R}^n$ *page 142*
- P_M - orthogonal projection of R^m onto M *page 144*
- underdetermined system of linear equations *page 145*
- H-inner product *page 149*
- H-norm *page 149*
- H-orthogonal vectors *page 149*
- H-orthogonal complement *page 149*
- H-generalized inverse*page 150*
- hyperplane H in $\mathbb{R}^n$ *page 158*
- boundary point of $C \subset \mathbb{R}^n$ *page 158*
- closure $\overline{A}$ of $A \subset \mathbb{R}^n$ *page 163*

Definitions (from Midterm 1, these may appear on Midterm 2):

- cosine of two vectors *page 6*
- distance of two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ *page 6*
- ball $B(\mathbf{x}, r)$ (what is $\mathbf{x}$ and r?) *page 6*
- interior D^0 of set $D \subseteq R^n$ *page 6, page 164*
- open set $D \subseteq \mathbb{R}^n$ *page 6*
- closed set $D \subseteq \mathbb{R}^n$ *page 7*
- compact set $D \subseteq \mathbb{R}^n$ *page 6*
- (global,local)(strict)minimizer and maximizer of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 8*
- critical point of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 8*
- gradient $\nabla f(\mathbf{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ *page 10*
- Hessian $Hf(\mathbf{x})$ where $f : \mathbb{R}^n \to \mathbb{R}$ *page 10*
- quadratic form associated with a symmetric matrix A *page 12*
- (positive,negative)(semi)definite matrix *page 13*
- indefinite matrix *page 13*
- saddle point of a function $f : \mathbb{R}^n \to \mathbb{R}$ *page 23*

- Δ_k, the k^{th} principal minor of a matrix A *page 16*
- coercive functions *page 25*
- eigenvalues and eigenvectors of a matrix A *page 29*
- convex sets in $\mathbb{R}^n$ *page 38*
- closed and open half-spaces in $\mathbb{R}^n$ *page 40*
- convex combination of k vectors from $\mathbb{R}^n$ *page 41*
- convex hull of $D \subseteq \mathbb{R}^n$ *page 42*
- (strictly) convex and concave function $f : C \to \mathbb{R}$, where $C \subseteq \mathbb{R}^n$ *page 49*

Theorems and statements (Midterm 2):

(*Try to not ignore assumptions - like if a function must be continuous etc.*)
(*Proofs are only for D14 (4 credit hour) students*)

- Describe transition form unconstrained geometric program to its dual using A-G inequality (pages 67, 68).
- How to compute a best least squares solution of a system $A\mathbf{x} = \mathbf{b}$? *Theorem 4.1.2*
- What is the QR factorization of a matrix A and when does it exists? *Theorem 4.1.5*
- If $M \subseteq \mathbb{R}^m$ factorization of a matrix A and when does it exists? *Theorem 4.1.5*
- How to compute P_M is a subspace and $x \in \mathbb{R}^m \setminus M$(orthogonal projection of R^m onto M) *Theorem 4.2.5*
- How to compute P_M? (orthogonal projection of $\mathbb{R}^m$ onto M) *Theorem 4.2.5*
- If $M \subseteq \mathbb{R}^m$ is a subspace, then what is $\left(M^{\perp}\right)^{\perp}$? *(Theorem 4.2.7)*
- What is the form of solutions of underdetermined systems? *Theorem 4.3.1, **with proof***
- What is the form of minimum norm solutions of underdetermined systems? *(Theorem 4.3.2, **with proof**)*
- What is the form of minimum H-norm solutions of underdetermined systems? *(Theorem 4.4.2)*
- If $C \subseteq \mathbb{R}^n$ is a convex set $y \in \mathbb{R}^n \setminus C$, then what is true if and only if $x^* \in C$ is the closest vector to y in C? *(Theorem 5.1.1)*
- What is the characterization of the closest vector of a convex set to a given vector using orthogonal complement? *(Theorem 5.1.2, **with proof**)*
- What is a sufficient condition for existence of a closest vector from a set C to a given vector $\mathbf{x}$? *(Theorem 5.1.3)*

Theorems and statements (from Midterm 1, these may appear on Midterm 2):

(*Try to not ignore assumptions - like if a function must be continuous etc.*)
(*Proofs are only for D14 (4 credit hour) students*)

- State Cauchy-Swartz inequality *(page 6)*
- Minimizers and maximizers of continuous function $f : I \to \mathbb{R}$ where $I \subset \mathbb{R}$ is a closed interval *(Theorem 1.1.4)*
- local minimizers and the gradient *(Theorem 1.2.3)*
- Taylor's formula for $\mathbb{R}^n$ *(Theorem 1.2.4)*

- H_f and global minimizers and maximizers? *(Theorem 1.2.5/Theorem 1.2.9)*
- Principal minors of matrix A and there relation to positive(negative) (semi)definite matrices A *(Theorem 1.3.3)*
- Eigenvalues of a symmetric matrix and there relation to positive(negative) (semi)definite matrices *(Theorem 1.5.1)*
- H_f and local minimizers and maximizers. *(Theorem 1.3.6,* ***with proof****)*
- coercive functions and minimization *(Theorem 1.4.4,* ***with proof****)*
- The convex hull of $D \subseteq \mathbb{R}^n$, $co(D)$, is the set of all convex combinations of vectors from D. ($D \subseteq \mathbb{R}^n$) *(Theorem 2.1.4)*
- convex function and continuity *(Theorem 2.3.1)*
- minimizers of convex functions *(Theorem 2.3.4* ***with proof****)*
- maximizers of concave functions *(Theorem 2.3.4)*
- the relationship between convex function and the gradient *(Theorem 2.3.5)*
- critical points of convex function and minimization *(Theorem 2.3.5 + Corollary 2.3.6)*
- The relationship between the Hessian and convexity of a function (in $\mathbb{R}^n$) *(Theorem 2.3.7)*
- building convex function from other convex functions *(Theorem 2.3.10)*
- inequality involving convex functions and convex combinations with the condition for equality (finite version of Jensen's Inequality) *(Theorem 2.3.3)*
- arithmetic-geometric mean inequality with the condition for equality *(Theorem 2.4.1* ***with proof****)*