Theorem 1. Let
\[
T = \begin{pmatrix} f & c^T \\ b & A \end{pmatrix}
\]
be a tableau, let \( B = (A_{j_1} \cdots A_{j_m}) \) with columns \( A_{j_1}, \ldots, A_{j_m} \) linearly independent and let \( X \) be a non-singular matrix with first column \( (1, 0, \ldots, 0)^T \). If
\[
XT = \tilde{T} = \begin{pmatrix} \tilde{f} & \tilde{c}^T \\ \tilde{b} & \tilde{A} \end{pmatrix}
\]
is such that
\[
\tilde{T} B = [\tilde{T}_{j_i} \cdots \tilde{T}_{j_m}] = \begin{pmatrix} 0^T \\ I \end{pmatrix},
\]
then
\[
\tilde{T} = \begin{pmatrix} f - \tilde{c}^T \overset{1}{\overset{-1}{B}} b & c^T - \tilde{c}^T \overset{1}{\overset{-1}{B}} A \\ \overset{1}{\overset{-1}{B}} b & \overset{1}{\overset{-1}{B}} A \end{pmatrix}
\]
and
\[
X = \begin{pmatrix} 1 & -\overset{1}{\overset{-1}{c}} B^{-1} \\ 0 & B^{-1} \end{pmatrix}
\]
where \( \overset{1}{\overset{-1}{c}} B = (c_{j_1}, \ldots, c_{j_m})^T \).

Remark.
An important corollary is that for each \( 1 \leq j \leq n \),
\[
(*) \quad \tilde{c}_j = c_j - \overset{1}{c} B^{-1} A_j
\]
and
\[
(*) \quad \tilde{A}_j = B^{-1} A_j.
\]

In the simplex procedure there are two row operations: either we multiple some row \( i \in \{1, \ldots, n\} \) by \( \alpha \), or we add some multiple of a row \( i \in \{1, \ldots, n\} \) to a row \( j \in \{0, \ldots, n\} \). The first operation corresponds to pre-multiplication of \( T \) by \( I(i, \alpha) \) which is obtained from the identity matrix \( I \) by replacing 1 in the entry \( (i, i) \) with \( \alpha \), and the second operation corresponds to pre-multiplication of \( T \) by \( I(i, j, \alpha) \) which is obtained from the identity matrix \( I \) by replacing 0 in the entry \( (i, j) \) with \( \alpha \). This means that \( \tilde{T} \) is obtained from \( T \) by a sequence of pre-multiplications by such matrices. Note that in all cases the matrix we pre-multiply by has the first column \( (1, 0, 0, \ldots, 0)^T \), i.e. we never add a multiple of row 0 to row \( i \) where \( i \in \{1, \ldots, n\} \) and we never multiply row 0 by a constant. Therefore, the product of these matrices has first column \( (1, 0, 0, \ldots, 0)^T \) since the product of two matrices with the zero column \( (1, 0, 0, \ldots, 0)^T \) also has the zero column \( (1, 0, 0, \ldots, 0)^T \).
Proof.

**Claim.** The inverse of a matrix $X$ with the first column $(1, 0, 0, \ldots, 0)^T$ also has the first column $(1, 0, 0, \ldots, 0)^T$.

Proof. Let $A$ be a matrix with $m$ columns. If $X$ has first column $(1, 0, 0, \ldots, 0)^T$, then the first column of $AX$ is simply the first column of $A$. Since $X^{-1}X = I$, the first column of $X^{-1}$ is the first column of $I$: $(1, 0, 0, \ldots, 0)^T$. □

Let $X = \begin{pmatrix} \frac{1}{0} \ u^T \ U \end{pmatrix}$ and by the claim we can let $X^{-1} = \begin{pmatrix} \frac{1}{0} \ w^T \ W \end{pmatrix}$. Note that $X^{-1}X = I$ implies that $0 = u^T + w^T U$ and $WU = I$. So $U = W^{-1}$ and $u^T = -w^T U$.

We have that $T = X^{-1}\bar{T}$. Therefore,

$$
\begin{pmatrix} c_B^T \\ B \end{pmatrix} = T_B = X^{-1}\bar{T}_B = \begin{pmatrix} \frac{1}{0} \ w^T \\ W \end{pmatrix} \begin{pmatrix} 0^T \\ I \end{pmatrix} = \begin{pmatrix} w^T \\ W \end{pmatrix}
$$

So $w = c_B$ and $W = B$. This implies $U = B^{-1}$ and $u^T = -c_B^T B^{-1}$. Now that we know $X$, we get the formula for $\bar{T}$, as well. □