Math 412 **HW9**

Due Friday, April 29, 2016

Students in the three credit hour course must solve five of the six problems. Students in the four credit hour course must solve all six problems.

1. Prove that if G is a color critical graph, then the graph G' generated from it by applying Mycielski's construction is also color critical.

2. Alternate proof of Turán's Theorem, including uniqueness

(a) Prove that a maximal simple graph with no $(r+1)$-clique has an r-clique.
(b) Prove that $e(T_{n,r}) = \binom{r}{2} + (n-r)(r-1) + e(T_{n-r,r})$.
(c) Use parts (a) and (b) to prove Turán's Theorem by induction on n, including the characterization of graphs achieving the bound.

3.

(a) Prove that $\chi(C_n; k) = (k-1)^n + (-1)^n(k-1)$
(b) For $H = G \vee K_1$, prove that $\chi(F; k) = k \cdot \chi(G; k-1)$. Using this and part (a), find the chromatic polynomial of the wheel $C_n \vee K_1$.

4. Let G be an n-vertex simple planar graph with girth k. Prove that G has at most $(n-2)\frac{k}{k-2}$ edges. Use this to prove that the Petersen graph is nonplanar.

5. Let G be a regular graph with a cut-vertex. Prove that $\chi'(G) > \Delta(G)$.

6. Let G be a Hamiltonian bipartite graph, and choose $x, y \in V(G)$. Prove that $G - x - y$ has a perfect matching if and only if x and y are on opposite sides of the bipartition of G. Apply this to prove that deleting two unit squares from an 8 by 8 chessboard leaves a board that can be partitioned into 1 by 2 rectangles if and only if the two missing squares have opposite colors.

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS: Section 6.1: # 1, 3, 4, 7, 8, 9, 10.
Section 6.3: # 1, 2 Section 7.1: # 1, 2, 4 Section 7.2: # 3

OTHER INTERESTING PROBLEMS: Section 6.1: # 18, 25, 27, 29.
Section 7.2: # 8, 12, 27