Math 412 **HW8**

Questions 1,2,3 - Due Friday, April 8, 2016, Questions 4,5,6 - Due Friday, April 15, 2016 (2 of 3 if three credit hour student)

Students in the three credit hour course must solve all three of the first three problems by 4/8/2016, and can solve any two of the three remaining problems by 4/15/2016.

Students in the four credit hour course must solve all of the first three problems by 4/8/2016, and must solve all of the three remaining problems by 4/15/2016.

1. Problem 4.3.2 in the book.

2. Use network flows to prove Menger's Theorem for internally-disjoint paths in digraphs: $\kappa(x, y) = \lambda(x, y)$ when xy is not an edge.

3. Suppose that there are p companies and that the ith company sends m_i representative to a conference. In this conference, there will be q work groups, each with at most n_j participants. The organizers of the conference want a schedule that assigns each representative to exactly one group, but no group can contain two representatives from the same company. The groups do not have to be filled to capacity.

(1) Show how to use network flows to test whether the constraints can be satisfied.
(2) Assume that $m_1 \geq \cdots \geq m_p$ and $n_1 \leq \cdots \leq n_q$. Prove that there exists the desired assignment of participants to groups if and only if, for all $0 \leq k \leq p$ and $0 \leq \ell \leq q$, it holds that $k(q - \ell) + \sum_{j=1}^{\ell} n_j \geq \sum_{i=1}^{k} m_i$.

4. Let G be a graph whose odd cycles are pairwise intersecting, meaning that every two odd cycles in G have a common vertex. Prove that $\chi(G) \leq 5$.

5. Prove that $\chi(G) = \omega(G)$ when $\overline{G}$ is bipartite. *(Hint: Phrase the claim in term of $\overline{G}$ and use a result related to bipartite graphs.)*

6. Prove that $\chi(G) + \chi(\overline{G}) \leq n(G) + 1$. *(Hint: Use induction on $n(G)$)*

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS: Section 4.3: # 1, 3. Section 5.1: # 1, 4, 7, 8, 12, 14, 15.

OTHER INTERESTING PROBLEMS: Section 4.3: # 5, 7, 10. Section 5.1: # 33, 38, 39, 40, 41.

Do not write these up!